Figure 2–1. Lunar Eclipse Contacts

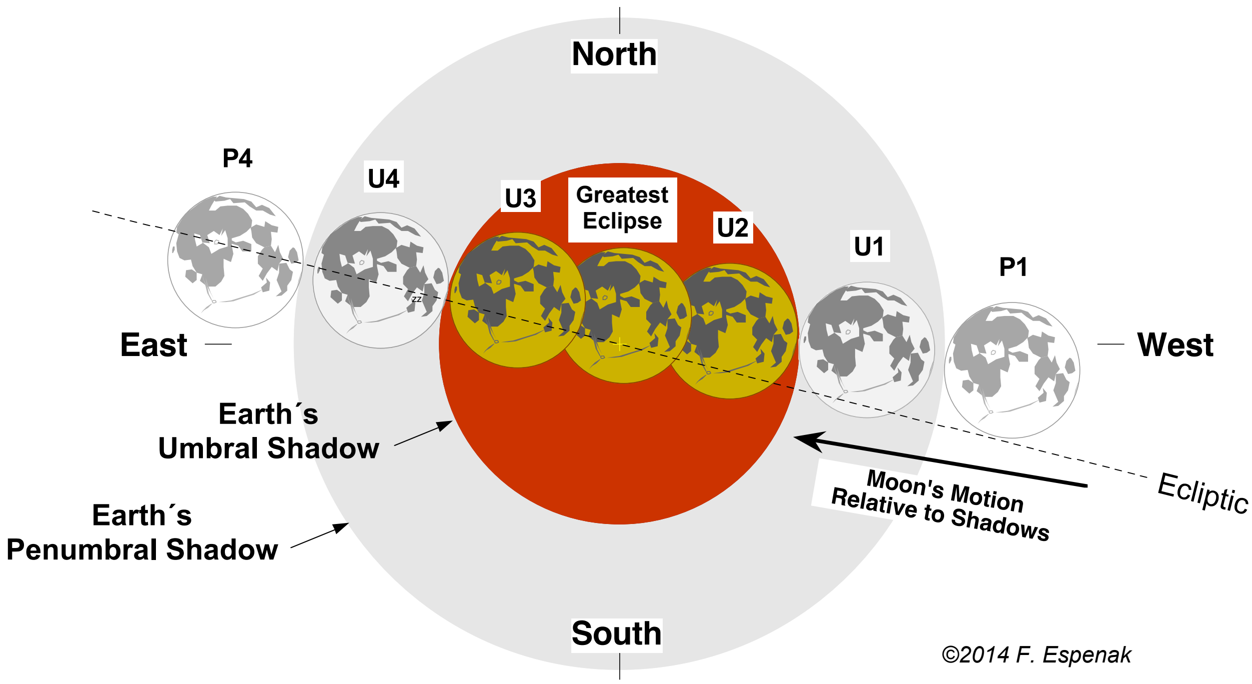

Figure 2–1 illustrates the six contacts for a total lunar eclipse. These correspond to the instants when the Moon's disk is externally tangent to the penumbra (P1 and P4), or either externally or internally tangent to the umbra (U1, U2, U3, and U4). Partial eclipses do not have contacts U2 and U3, while penumbral eclipses only have contacts P1 and P4.

2.2 Enlargement of Earth's Shadows

In 1707, Philippe de La Hire made a curious observation about Earth's umbra. The predicted radius of the shadow needed to be enlarged by about 1/41 in order to fit timings made during a lunar eclipse. Additional observations over the next two centuries revealed that the shadow enlargement was somewhat variable from one eclipse to the next. According to William Chauvenet (1891):

"This fractional increase of the breath of the shadow was given by Lambert as 1/40, and by Mayer as 1/60. Beer and Maedler found 1/50 from a number of observations of eclipses of lunar spots in the very favorable eclipse of December 26, 1833."

Chauvenet adopted a value of 1/50, which has become the standard enlargement factor for lunar eclipse predictions published by many national institutes worldwide. The enlargement enters into the definitions of the penumbral and umbral shadow radii as follows.

penumbral radius: $$Rp = 1.02 \times (0.998340 \times \pi m + Ss + \pi s) \quad (2\text{–}1)$$

umbral radius: $$Ru = 1.02 \times (0.998340 \times \pi m - Ss + \pi s) \quad (2\text{–}2)$$

Where: πm = Equatorial horizontal parallax of the Moon

Ss = Geocentric semi-diameter of the Sun

πs = Equatorial horizontal parallax of the Sun

The factor 1.02 is the enlargement of the shadows by 1/50. Earth's true figure approximates that of an oblate ellipsoid with a flattening of ~1/300. Furthermore, the degree of axial tilt of the planet towards or away from the Sun throughout the year means the shape of the penumbral and umbral shadows varies by a small amount. A mean radius of Earth at latitude 45° is used to approximate the departure from perfectly circular shadows. The

Astronomical Almanac uses a factor of 0.998340 to scale the Moon's equatorial horizontal parallax to account for this (0.998340 ≈ 1.0 – 0.5 × 1/300).

Some authorities dispute Chauvenet's shadow enlargement convention. Danjon (1951) notes the only reasonable way of accounting for a layer of opaque air surrounding Earth is to increase the planet's radius by the altitude of the layer. This is accomplished by proportionally increasing the parallax of the Moon. The radii of the umbral and penumbral shadows are then subject to the same absolute correction and not the same relative correction employed in the traditional Chauvenet 1/50 convention. Danjon estimates the thickness of the occulting layer to be 75 kilometers and this results in an enlargement of Earth's radius and the Moon's parallax of about 1/85.

In 1951, the French almanac *Connaissance des Temps* adopted Danjon's method for the enlargement of Earth's shadows in their eclipse predictions as shown below.

Penumbral radius: $Rp = 1.01 \times \pi m + Ss + \pi s$ (2–3)

Umbral radius: $Ru = 1.01 \times \pi m - Ss + \pi s$ (2–4)

Where: πm = Equatorial horizontal parallax of the Moon
Ss = Geocentric semi-diameter of the Sun
πs = Equatorial horizontal parallax of the Sun

And $1.01 \approx 1 + 1/85 - 1/594$

The factor 1.01 combines the 1/85 shadow enlargement term with a 1/594 term to correct for Earth's oblateness at a latitude of 45°.

Danjon's method correctly models the geometric relationship between an enlargement of Earth's radius and the corresponding increase in the size of its shadows. Meeus and Mucke (1979) and Espenak & Meeus (2009a) both use Danjon's method. However, the resulting umbral and penumbral eclipse magnitudes are smaller by approximately 0.006 and 0.026, respectively, as compared to predictions using the traditional Chauvenet convention of 1/50.

For instance, the umbral magnitude of the partial lunar eclipse of 2008 Aug 16 was 0.813 according to the *Astronomical Almanac for 2008* (2008) using Chauvenet's method, but only 0.8076 according to Espenak & Meeus (2009a) using Danjon's method.

Chauvenet's method is still used by the *Astronomical Almanac* (published jointly by the USNO and HMNAO) to calculate lunar eclipse circumstances, while Danjon's method is used by Meeus and Mucke (1979), Espenak and Meeus (2009a) and *Connaissance des Temps* (published by the Bureau des Longitudes).

2.3 Earth's Elliptical Shadows

Both the Chaunenet and Danjon methods of accounting for the enlargement of Earth's two shadows assume circular shadows scaled at 45° latitude. However, Earth is flattened at the poles and bulges at the Equator, so an oblate spheroid more closely represents its shape. The projection of each of the planet's shadows is an ellipse rather than a circle. Furthermore, Earth's axial tilt towards or away from the Sun throughout the year means the elliptical shape of the penumbral and umbral shadows varies as well.

Herald and Sinnott performed an analysis of 22,539 observations made at 94 lunar eclipses from 1842 to 2011 (Herald and Sinnott, 2014). This is the largest collection of crater and contact timings ever compiled. The authors define the height of a 'notional eclipse-forming layer' in Earth's atmosphere (abbreviated as NEL) corresponding to the occulting layer height used by Danjon. Given the size and consistency of their dataset, they refine the NEL height to 87 kilometers (compared to Danjon's value of 75 kilometers).

Herald and Sinnott find that size and shape of the umbra are consistent with an oblate spheroid at the time of each eclipse, enlarged by the empirically determined NEL that uniformly surrounds Earth. They conclude that future lunar eclipse predictions should be based on a Danjon-like approach with full allowance for an oblate Earth, with the umbral radius r_u being computed using equation 2–5.

$$r_u = R_\oplus \, \pi_m - S_s + \pi_s \tag{2–5}$$

Where: $R_\oplus$ = Radius of Earth, where $R_\oplus = 1 + h - 0.003353 \sin^2\psi \cos^2(\delta_s + f \sin\psi)$

h = 0.0136 is the height of the NEL in Earth radii (h = 87 / 6378.137)

ψ = Angular position angle (measured from the east–west direction, positive to the north) of the relevant contact point about the edge of the umbra

$f = S_s - R_\oplus \, \pi_s$

δ_s = Declination of the Sun

π_m = Equatorial horizontal parallax of the Moon

S_s = Geocentric semi-diameter of the Sun

π_s = Equatorial horizontal parallax of the Sun

The calculation of r_u requires a single iteration between $R_\oplus$ and f to generate mutually consistent values for a given ψ. Similar adjustments can be made for the penumbral radius r_p although the resulting effects are not observable.

The Herald and Sinnott method of calculating Earth's shadow enlargement is the most rigorous and accurate procedure to date. It is superior to the methods of Chaunenet and Danjon because it uses a better determined value of the NEL and an elliptical cross section for Earth's shadow. The *21st Century Canon of Lunar Eclipses* uses the Herald and Sinnott method in the lunar eclipse predictions presented here.

2.4 Solar and Lunar Coordinates

The coordinates of the Sun and the Moon used in the eclipse predictions presented here have been calculated with the JPL DE430 (Jet Propulsion Laboratory Developmental Ephemeris 430). The DE430 is based upon the International Celestial Reference Frame (ICRF), the adopted reference frame of the IAU. The DE430 includes both nutation or libration and has an absolute accuracy of several kilometers for planetary positions. In most cases this corresponds to a small fraction of an arc-second.

The Moon's center of figure does not coincide with its center of mass. To compensate, an empirical correction is sometimes added to the Moon's center of mass position. Unfortunately, the large variation in lunar libration from one eclipse to the next minimizes the effectiveness of this empirical correction. Because of this, no correction has been made to the Moon's center of mass position in the *21st Century Canon*.

2.5 Measurement of Time

The most natural form of time measurement is the solar day (usually measured from solar noon to solar noon). The length of the solar day varies during the year because of the eccentricity of Earth's orbit around the Sun. Mean solar time resolves this problem by using an average to define the mean solar day.

In 1884, Greenwich Mean Time (GMT) — the mean solar time on the Greenwich Meridian (0° longitude) — was adopted as the standard reference time for clocks around the world. A fundamental basis of GMT is the assumption that Earth's rotation on its axis is constant. It wasn't until the mid-twentieth century that astronomers realized the rotation period is gradually increasing. Earth is slowing down because of tidal friction with the Moon.

For purposes of orbital calculations, time using Earth's rotation was abandoned for a more uniform time scale based on Earth's orbit about the Sun. In 1952, Ephemeris Time was introduced to address the problem. The ephemeris